1 Arithmetic sequence

In an Arithmetic Sequence the difference between one term and the next is a constant.

1st, 2nd, 3rd, 4th, ...
3, 8, 13, 18, ...

How to find patterns;
Goal is to have an equation defining the rth term by 1st term and r.

1. Look at the first term: 3
2. Check the difference between one term and the next: 5
3. Calculate the next term by using the first term and the order placement (which is a counting number series) : $8 = 3 + (\boxed{2} - 1) \times 5$ where 2 means the second term.
4. Double check to get 3rd or 4th term by replacing only the order number above.
   Ex. For the 3rd term: $3 + (\boxed{3} - 1) \times 5 = 13$
   Ex. For the 4th term: $3 + (\boxed{4} - 1) \times 5 = 18$
5. Only first term, the difference, and the placement will tell you any term in the sequence.

General formula for the nth term:

$$a_n = a_1 + d \times (n - 1) \tag{1}$$

Check: This formula must be able to apply to a_1, the first term.

$$a_1 = a_1 + d \times (1 - 1) = 3 + 5 \times 0 = 3 \tag{2}$$

Sum of the first n terms:

$$\begin{aligned} \Sigma &= \frac{1}{2}(a_1 + a_n) \times n = \frac{1}{2}[a_1 + a_1 + d \times (n - 1)] \times n \\ \Sigma &= \frac{1}{2}n[2a_1 + d \times (n - 1)] \end{aligned} \tag{3}$$

Q 1. What is the sum of $1, 2, 3, 4, \cdots, 10$? What is the sum from 1 to 100?

1, 2, 3, 4, ..., 10
10, 9, 8, 7, ..., 1

Inverse the series, and assume the sum is S. Then:

$$2S = \boxed{10} \times (1 + 10) = \boxed{10} \times 11 = 110$$
$$S = \frac{1}{2}(\boxed{10} \times (1 + 10)) = 55$$

Q 2. What is the sum of $1, 2, 3, 4, \cdots, 100$

Q 3. What is the sum of $2, 4, \cdots, 10$

Q 4. What is the sum of $1, 3, 5, \cdots, 10$

Q 5. What is the sum of $3, 8, 13, 18, 23$

$3, 8, 13, 18, 23$
$23, 18, 13, 8, 3$

Reference: Arithmetic Sequence

2 Pratice

Q 1. What is the 51th term in Natural Number Series {1, 2, 3, ...} find the sum of the first 51 numbers.

Q 2. What is the 51th term in Even Number Series {2, 4, 6, ...} find the sum of the first 51 numbers.

Q 3. What is the 51th term in Odd Number Series {1, 3, 5, ...} ; find the sum of the first 51 numbers.

Q 4. Find three consecutive numbers whose sum is 72. Assuming n is the middle number.

Q 5. If n represents the counting number set $\{1, 2, 3, 4, 5, \cdots\}$
What can represent the even number set $\{2, 4, 6, 8, 10, \cdots\}$
What can represent the odd number set $\{1, 3, 5, 7, 9, \cdots\}$

Q 6. Find three consecutive even numbers whose sum is 72.

Q 7. Find three consecutive odd numbers whose sum is 69.

Q 8. What is the nth term in Natural Number Series {1, 5, 9, 13, 17, ...} find the sum of the first n numbers.
Note: $4n - 3$ where $a_0 = -3$; or $4(n - 1) + 1$ where $a_1 = 1$.

Q 9. What is the nth term in Natural Number Series {6, 10, 14, 18, 22, ...} find the sum of the first n numbers.

Q 10. What is the nth term in Natural Number Series {6, 14, 22, 30 ...} find the sum of the first n numbers.

Q 11. What is the nth term in Natural Number Series {4, 8, 12, 16, ...} find the sum of the first n numbers.

Q 12. What is the nth term in Natural Number Series {3, 6, 9, 12, ...} find the sum of the first n numbers.

3 Geometric sequence

Q 1. Picture a cat stalking a mouse. They're about 100 inches apart. Every time the mouse starts nibbling at the hunk of cheese, the cat takes advantage of the mouse's distraction and creeps closer by one-tenth the distance between them. The cat wants to get about 6 inches away - close enough to pounce. How far apart are they after four moves? How about after 10 moves? How long will it take before the cat can pounce on the mouse?
Mouse ——— 100 inches————Cat
Mouse ——- 90 inches —— Cat
Mouse ——81 inches — Cat
Mouse — 72.9 inches – Cat
Mouse – 65.61 inches - Cat

$$100 \times (1 - \frac{9}{10})^n \text{ where (n=4, 10)}$$

$$100 \times (1 - \frac{9}{10})^n \approx 100 - 6$$

Q 2. Find the total distance that a super ball travels if it always bounces back 75 percent of the distance it fell. You dropped it from a window that's 40 feet above a nice smooth sidewalk. Assume that the ball always falls straight down and returns straight up.

$$40, 40 \times 75\%, 40 \times 75\% \times 75\%, 40 \times 75\% \times 75\% \times 75\%, ...$$

$$\begin{aligned} & 40 + 40 \times 75\% + 40 \times (75\%)^2 + 40 \times (75\%)^3 + ... \\ =& 40 \times (1 + 75\% + (75\%)^2 + (75\%)^3 + ...) \end{aligned}$$

$$A \times (1 - r) = A - A \times r.. \text{ (Distributive property)}$$

$$\begin{aligned} & (1 + r + r^2 + r^3 + ...) \times (1 - r) \\ =& (1 + r + r^2 + r^3 + ...) - r(1 + r + r^2 + r^3 + ...) \\ =& (1 + r + r^2 + r^3 + ...) - r - r^2 - r^3 - ... \\ =& 1 \end{aligned}$$

$$\Rightarrow (1 + r + r^2 + r^3 + ...) = \frac{1}{1-r}$$

$$\begin{aligned} & (1 + r + r^2 + r^3 + ... + r^{n-1}) \times (1 - r) \\ =& (1 + r + r^2 + r^3 + ... + r^{n-1}) - r(1 + r + r^2 + r^3 + ...r^{n-1}) \\ =& (1 + r + r^2 + r^3 + ... + r^{n-1}) - r - r^2 - r^3 - ... - r^n \end{aligned}$$

$$\begin{aligned} =& \ 1 \\ \Rightarrow & (1 + r + r^2 + r^3 + ... + r^{n-1}) = \frac{1-r^n}{1-r} \end{aligned}$$

Definition 1. **A geometric sequence** is formed when each term is found by multiplying the previous term by a particular number, called the ratio.

- The sum of the terms of an infinite geometric sequence:
  $Sum = \frac{a}{1-r}$. where $0 < r < 1$
- The sum of the first n terms of any geometric sequence:
  $Sum = \frac{a \times (1-r^n)}{1-r}$. where $0 < r < 1$