1 Focuses: Chapter 3

1. Solve for equations: align each = so that to keep both sides in balance
2. Order: depends on situation to follow different orders; goal is to keep balance in check; and to simplify the problem; unknown letter is on the left; number terms on the right.
   (a) Remove parenthesis by using distributive property (including inverse of the sum)
   (b) Insert parenthesis by factoring
   (c) sort and collect the like terms
   (d) add/subtract same term
   (e) multiply/divide same term
3. Equation with fractions, decimals, ratio/proportions, and percent
4. Equation with absolute value

2 The order of operations – PEMDAS

2.1 Meaning of each operation and the math fact families

1. Addition and Subtraction:
   $A + B = B + A = C$
   $C - A = B$
   $C - B = A$

2. Multiplication and Division:
   $A \cdot B = B \cdot A = C$
   $C \div A = \frac{C}{A} = B$
   $C \div B = \frac{C}{B} = A$

3. Exponent and Radical
   When n is an odd number, $A = B^n \quad \sqrt[n]{A} = B$
   Ex1. $27 = 3^3 \ \sqrt[3]{27} = 3$
   $Ex2.$ - $27 = (-3)^3 \ \sqrt[3]{-27} = -3$

   When n is even number: $A = B^n \quad \sqrt[n]{A} = \pm B$
   Ex. $9 = 3^2 = (-3)^2 \ \sqrt{9} = \pm 3$

4. Two kinds of cancel out
   Additive inverse: $a + (-a) = 0$
   Multiplicative inverse $a \cdot \frac{1}{a} = a \div a = 1$

5. Absolute value $|9| = |-9| = 9$

2.2 Grouping symbols – P

1. Parentheses (..)
2. Brackets [..] {..}
3. Radical $\sqrt{...}$
4. Fraction line (——)
5. Absolute value |..|

If there are more than 2 layers of grouping symbols,

1. Look for all the pairs and symbols' starting point and ending point, work on the innermost first,
2. Then from inside to outside.

2.3 Without or within grouping symbols

1. Exponents: Powers and roots
2. Multiplication and division
3. Addition and subtraction

For any rational numbers $a, b, andc$

- The Addition Property of Equality
  if $a = b$, then $a + c = b + c$
- The Multiplication Property of Equality
  if $a = b$, then $ac = bc$
- Special Formulas
  - Average of three numbers: $A = \frac{a+b+c}{3}$
  - $3A = a + b + c$
  - $a = 3A - b - c$
  - Perimeters and areas of certain shapes
- Absolute Value

$$|x| = a$$
$$x = a \ \textbf{OR} \ x = -a$$

- fractions, ratios, proportions, percents

3 Remove or Insert parentheses

3.1 Distributive property vs factoring

$$a \times (b + c) = ab + ac \textbf{ distributing} \tag{1}$$

$$ab + ac = a \times (b + c) \textbf{ factoring} \tag{2}$$

3.2 Inverse of a sum

$$-1 \cdot a \iff -a \tag{3}$$

$$-1 \cdot (a + b) = -a + (-b) = -a - b$$

$$-1 \cdot (a + b) \iff -a - b \tag{4}$$

$$\begin{aligned} -1 \cdot (a - b) &= -1 \cdot [a + (-b)] \\ &= (-1) \times (a) + (-1) \times (-b) \\ &= -a + b \end{aligned}$$

$$-1 \cdot (a - b) \iff -a + b \tag{5}$$

3.3 Inverse of a product

$$1 \div a \iff \frac{1}{a} \tag{6}$$

$$1 \div (a \cdot b) = \frac{1}{ab} = 1 \div a \div b$$

$$1 \div (a \cdot b) \iff 1 \div a \div b \tag{7}$$

$$\begin{aligned} 1 \div (a \div b) &= 1 \div \frac{a}{b} \\ &= 1 \times \frac{b}{a} \\ &= 1 \times \frac{1}{a} \times b \\ &= 1 \div a \cdot b \end{aligned}$$

$$\begin{aligned} 1 \div (a \div b) &= \frac{1}{\frac{a}{b}} \\ 4 &= \frac{1 \times b}{\frac{a}{b} \times b} \\ &= \frac{b}{a} \\ &= 1 \div a \cdot b \end{aligned}$$

$$1 \div (a \div b) \iff 1 \div a \cdot b \tag{8}$$

4 Checking your answers

1. Does the answer make any sense?
2. Does it work? That is, putting the answer back into the problem gives you a true statement.