# Ratio and Proportion

PH 3-6, 3-9, 3-10

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## 1 Division - Fraction - Ratio - Percent - Decimal

$$3 \div 12 = \frac{3}{12} = 3 : 12 = 25\% = 0.25 \tag{1}$$

## 2 Ratios

**Definition 1.** **A ratio** of two quantities is a comparison, often expressed as a fraction.

For example: the ratio of the age of the mother to that of the daughter is 49 to 10 or $\frac{49}{10}$. The ratio of 49 to 12 can be expressed in several ways:

$$49 : 10$$

$$49 \div 10$$

$$\frac{49}{10}$$

$$4.9$$

- To compare ratios, you need to convert them into fractions
- A ratio can be: part to part; part to whole; whole to part.

Method: Translate words into math as you read. Or draw a chart and mark the math information on the chart. Try **read only once** to get all the math information.

**Q 1.** Which ratio is larger 1:2 or 4:5?

**Q 2.** $9m = 5n$ Find the ratio $m : n$

**Q 3.** Total length of a rocket is 180 ft. It has three parts: top - cargo/navigation; middle - fuel tank; bottom - the booster; the ratio between these three parts are: 1:5:6. What is the height of each part?

**Q 4.** Group 20 people A) into two groups based on the ratio 2:3 and then B) into 3 groups based on the ratio 6:3:1. How many people in each group in A) and B)?

**Q 5.** Among 20 people. The ratio between male and female is 2:3. How many males and how many females?

**Q 6.** Suppose you are baking 24 cookies, you need 2 cups of flour, 3 cups of water, 1/2 cup of sugar. Then the ratio or the proportion of your ingredients are

$$2 : 3 : \frac{1}{2}$$

The ratio of flour and water is: 2:3, 2 to 3, or $\frac{2}{3}$
How many cups of flour and water you need if you want to bake 48 cookies?

# 3 Proportions

**Definition 1.** An equation that states that two ratios are equal is called **a proportion**.

For example: $\frac{2}{3} = \frac{6}{9}$ ; $\frac{x}{24} = \frac{2}{3}$

Two things are in proportion when a change in one cause a related change in the other. There are two kinds of changes as follow

1. the ratio stays same: the two things goes up or down proportionally
2. the product stays the same: one thing goes up while the other goes down by the same proportion

**Q 1.** Diego hosted a spaghetti dinner for the soccer team. He made 6 boxes of spaghetti to feed the 20 people that came. Next time, 50 people are coming! How many boxes of spaghetti should Diego buy to feed all those people? Explain or show your reasoning.

**Q 2.** You are sketching a person with different rectangles. You want the height of the head and body part to be 1:7. If the person you draw is 40 cm on you paper, how tall is the body part?

**Q 3.** Each gardener plants 2 trees a day. How many trees can 5 gardeners plant? (**the ratio stays same: the two things goes up or down proportionally**)

**Q 4.** If 3 bags contain 18 apples. How many apples do 5 bags contain?

**Q 5.** It takes one van 8 days to deliver some parcels; 2 vans 4 days. How many days will it take if there are 3 vans. 1:8, 2:4; 3:? (**the product stays the same: one thing goes up while the other goes down by the same proportion**)

**Q 6.** Write an equation to represent each situation and then solve the equation.

a. Andre drinks 15 ounces of water, which is $\frac{3}{5}$ of a bottle. How much does the bottle hold? Use x for the number of ounces of water the bottle holds.

b. A bottle holds 15 ounces of water. Jada drank 8.5 ounces of water. How many ounces of water are left in the bottle? Use y for the number of ounces of water left in the bottle.

c. A bottle holds z ounces of water. A second bottle holds 16 ounces, which is $\frac{8}{5}$ times as much water. How much does the first bottle hold?

**Q 7.** There are 15.24 centimeters in 6 inches.

a. How many centimeters are in 1 foot? (1 foot = 12 inches)

b. How many centimeters are in 1 yard (1 yard = 3 feet)?

**Q 8.** Relate: What operation(s) you can use to compare two things?

# 4 Scale of shape and map

## 4.1 Scaling

Scaling is making something larger or smaller while keeping everything in the same proportion - which means making all parts larger or smaller by the same amount.

We can use scaling to change numbers, amounts, or the sizes of objects or shapes.

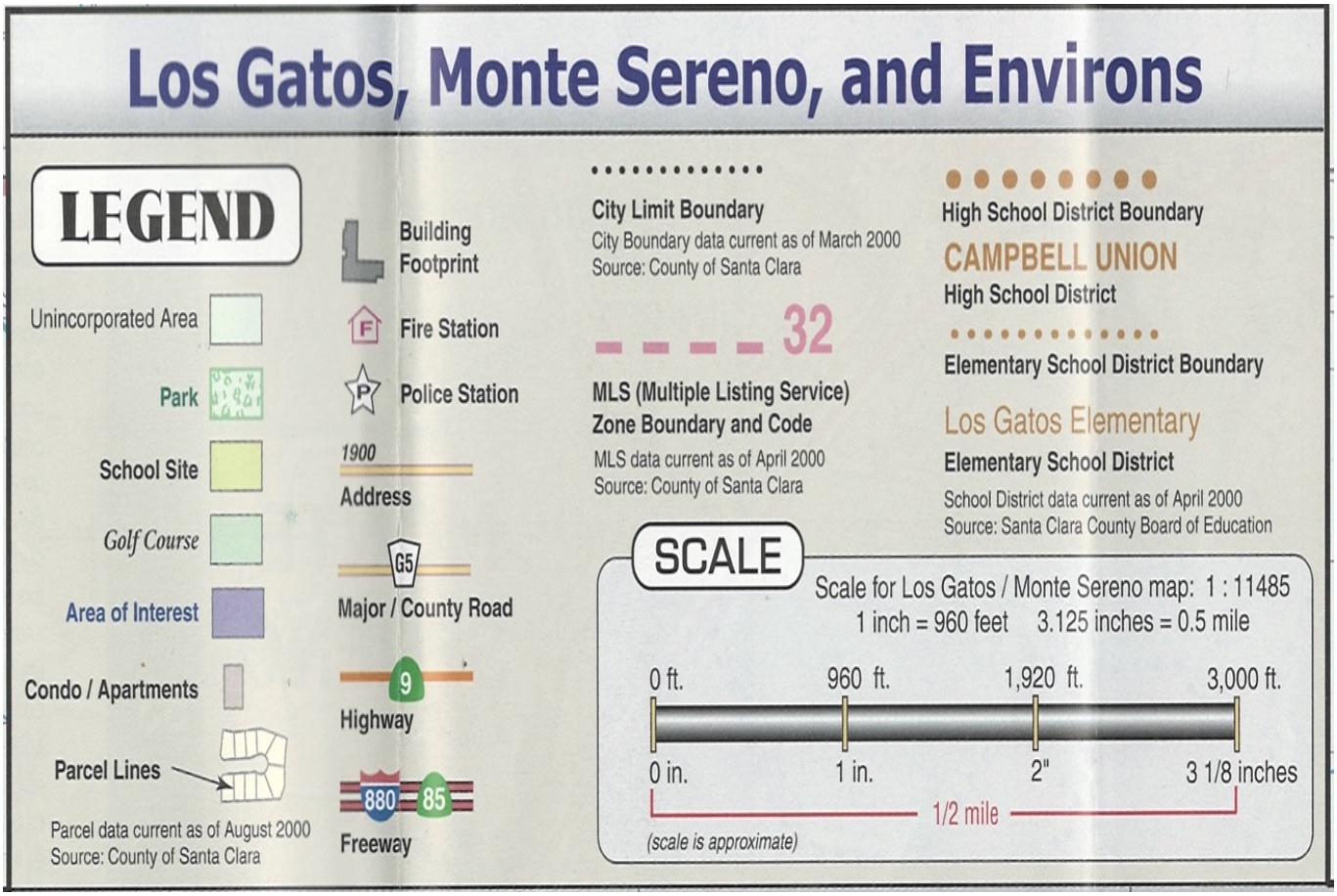
1. Scaling is based on size - or one dimension length only - length, width, perimeter; scaled up by 2 times
2. area is two dimensional concept; scaled up by $2^2 = 4$ times; area change is squared
3. volume is three dimensional; scaled up by $2^3 = 8$ times; volume change is cubic.
4. the angles stay same all the time.

When you enlarge a shape, for example, a rectangular of $a$ cm long and $b$ cm wide into a new one which is two times as large as the original one. The new rectangular will be of $2a$ cm long and $2b$ cm wide.

$Areaofnewrectangle = 2a \times 2b = 4absquaredcm$ which is 4 times as large as the original area.

The volume of Right Rectangular Prism of $hcm$ high $= 2a \times 2b \times 2h = 8abh$ which is $2^3 = 8$ times as large as the original one.

## 4.2 Map Scales



# 5 Important Notes

- **Units**: Usually, the measurement unit should be the same. But if you are very clear about the unit for each part, like a map's scale is 1 cm : 100 km, it is OK as well. Whether to use the same unit or to use different unit, it depends on whichever is easy or you are comfortable with. But when you need to calculate or compare, you need to convert everything into same unit.
- **Meaning of each number**: corresponding part or whole. For example: in the above map: 1:11485; 1 refers to the distance on the map while 11485 refers to the actual distance. That is, 1 cm on the map equals to 11485 cm in the real world.
- **Larger or smaller scale**: usually we use "1 (on the map) : x real distance" because we use map to figure out the real distance. Surveyors who draw maps will do the opposite. Thus, the smaller the ratio (larger denominator or second number), the less detail the map. Therefore, a map or a drawing of a real place, such as a pool, will be smaller if it uses a smaller scale.