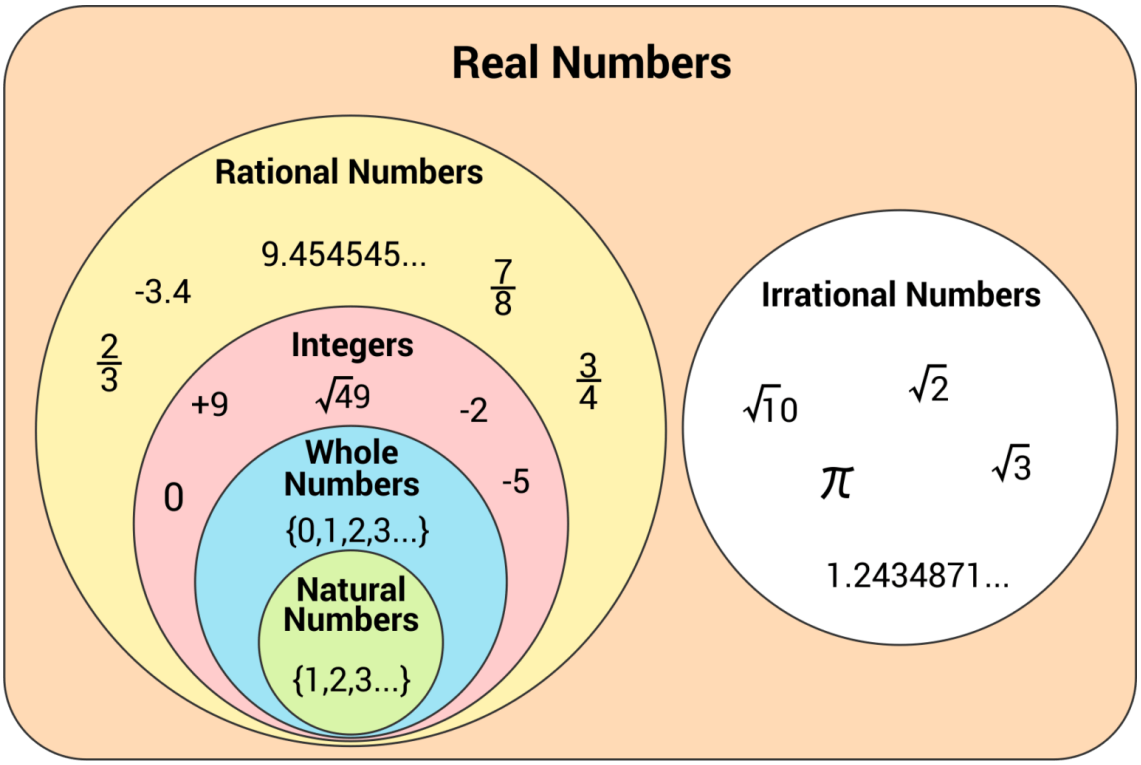
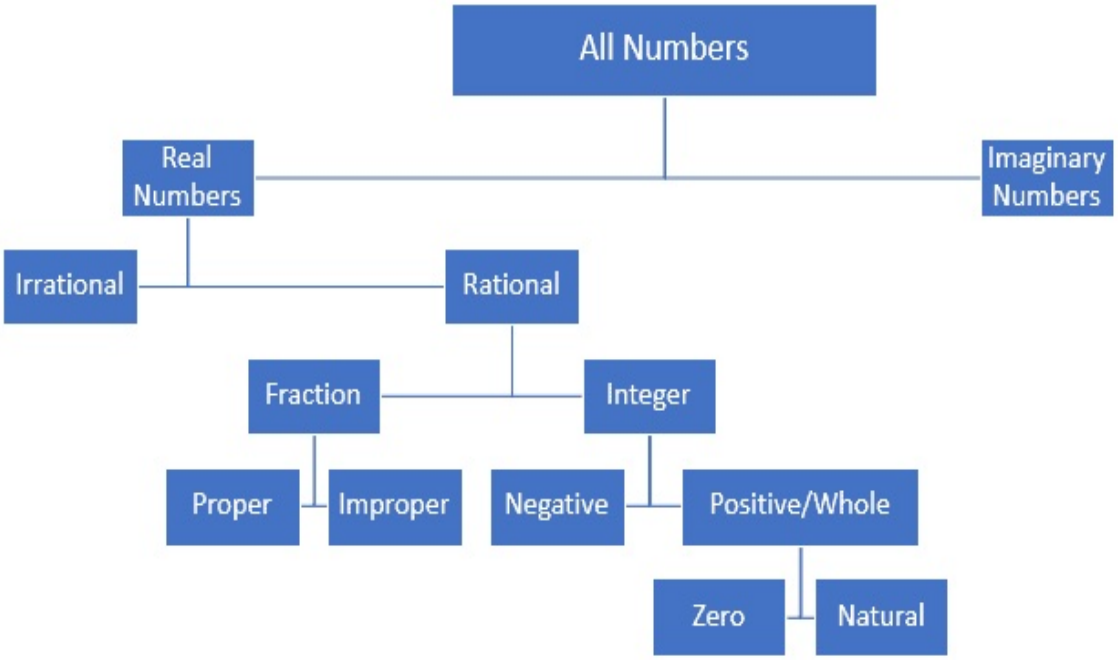
Numbers & Operations and Their Properties

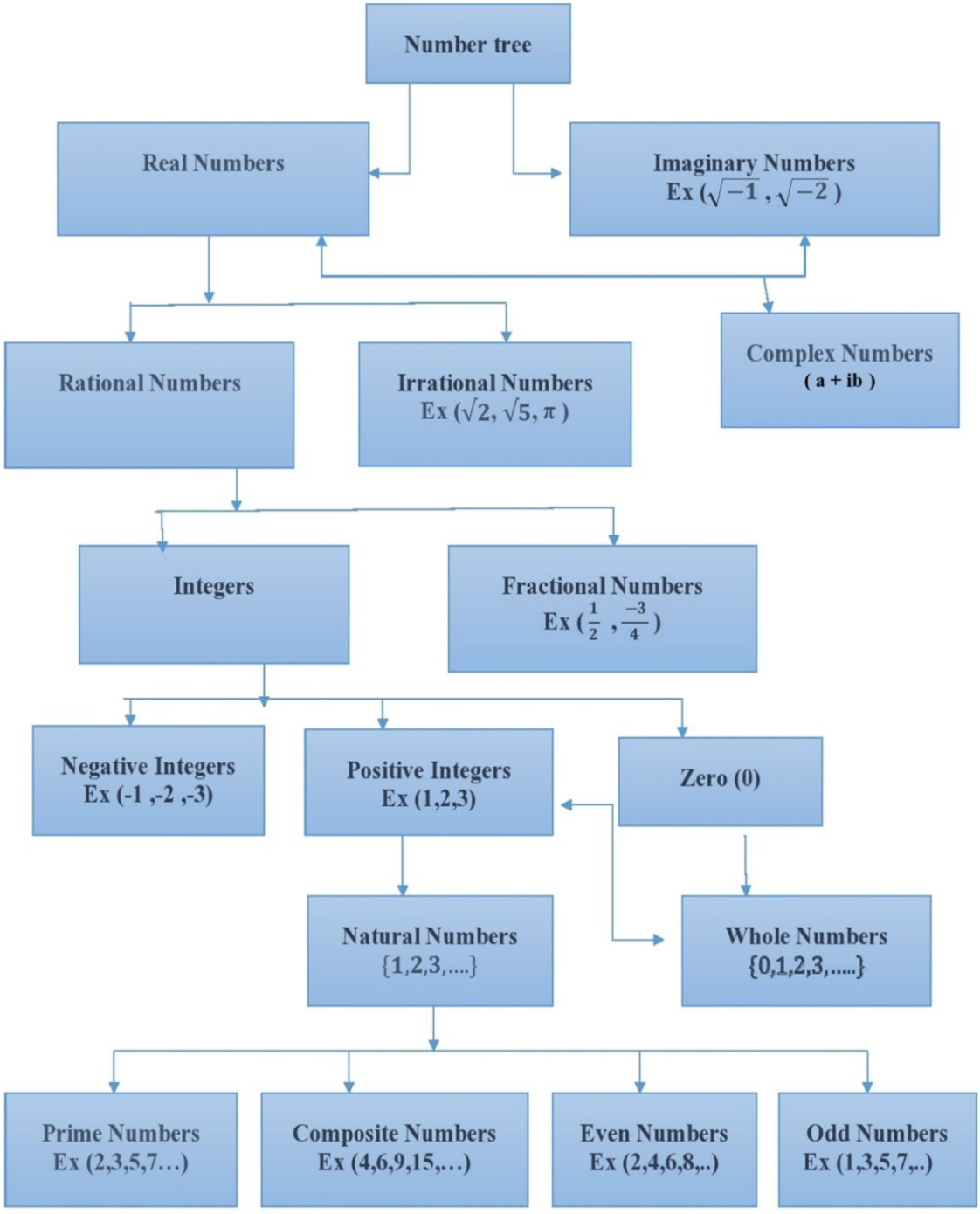
PH: Chapter 1 & 2

November 3, 2021

1 Classification of Numbers




All Numbers
Real Numbers
Imaginary Numbers
Irrational
Rational
Fraction
Integer
Proper
Improper
Negative
Positive/Whole
Zero
Natural


Number tree
Real Numbers
Imaginary Numbers
Ex ($\sqrt{-1}$, $\sqrt{-2}$)
Rational Numbers
Irrational Numbers
Ex ($\sqrt{2}$, $\sqrt{5}$, π)
Complex Numbers
(a + ib)
Integers
Fractional Numbers
Ex ($\frac{1}{2}$, $\frac{-3}{4}$)
Negative Integers
Ex (-1 ,-2 ,-3)
Positive Integers
Ex (1,2,3)
Zero (0)
Natural Numbers
{1,2,3,....}
Whole Numbers
{0,1,2,3,.....}
Prime Numbers
Ex (2,3,5,7…)
Composite Numbers
Ex (4,6,9,15,…)
Even Numbers
Ex (2,4,6,8,..)
Odd Numbers
Ex (1,3,5,7,..)

2 Properties of Rational Numbers

For any rational numbers $a, b, andc$

- The Closure Properties
  - 1. Addition: $a + b$ is a rational number
  - 2. Multiplication: ab is a rational number
- The Commutative Properties
  - 3. Addition: $a + b = b + a$
  - 4. Multiplication: $ab = ba$
- Associative Properties
  - 5. Addition: $a + (b + c) = (a + b) + c$
  - 6. Multiplication: $a(bc) = (ab)c$
- Identity Properties
  - 7. Addition: $a + 0 = a$
  - 8. Multiplication: $a \cdot 1 = a$
- Inverse Properties
  - 9. Addition: $a + (-a) = 0$
  - 10. Multiplication: $a \cdot \frac{1}{a} = 1$
- Distributive Property of Multiplication over Addition
  - 11. $a(b + c) = ab + ac$
- Inverse of Sum
  - 12. $-1 \cdot a = -a$
  - 13. $-1 \cdot (a + b) = -a + (-b) = -a - b$
  - 14.

$$\begin{aligned} -1 \cdot (a - b) &= -1 \cdot [a + (-b)] \\ &= (-1) \times (a) + (-1) \times (-b) \\ &= -a + b \end{aligned}$$

3 Notes on Numbers

- Rational numbers: any number that can be represented as $\frac{a}{b}$ where $b \neq 0$
- Counting numbers: natural numbers, positive whole numbers
- Consecutive numbers (n-1, n, n+1); Consecutive odd numbers: Consecutive even numbers: ...

Q 1. Why 0 is special?

- Any number added by 0 remains that number, as 0 is the identity for addition.
- $0 \cdot a = 0$ (where $a \neq 0$)
- Fraction: $\frac{0}{a} = 0$; 0 cannot be denominator
- Exponent: $a^0 = 1$

Q 2. Why 1 is special?

- One, sometimes referred to as unity,[4][1] is the first non-zero natural number. It is thus the integer after zero.
- Any number multiplied by one remains that number, as one is the identity for multiplication.
- 1 is its own factorial, its own square and square root, its own cube and cube root, and so on.
- It is also the only natural number that is neither composite nor prime with respect to division, but is instead considered a unit
- Fraction: 1 is whole, as you can divide a whole into any number of parts. And you can use $\frac{a}{a} = 1$ to get many equivalent fractions. $a = \frac{a}{1}$; $a \times \frac{1}{a} = 1$
- Exponent: $a^0 = 1$

4 Relationships between four basic operations and numbers

- +: positive number
- -: negative number
- x: a number's prime factorization; powers, and roots
- ÷: whole number, fraction, ratio, percent, decimal

5 Axioms (Properties) for Rational Numbers or Operations

5.1 Inverse Properties

Condition: All four operations
Action: Each operation has an inverse - an operation that undoes it.

5.2 Commutative Properties

We can change the order when adding without affecting the sum.
$a + b = b + a$

We can change the order when multiplying without affecting the product.
$a \cdot b = b \cdot a$

Condition: Addition and multiplication; for two numbers or terms
Action: **switch**.
Question: How about subtraction and division? (hint: can you turn a subtraction to addition and a division to a multiplication?)

$$5 + 3 = 8 \tag{1}$$

$$3 + 5 = 8 \tag{2}$$

$$2 \times 7 = 14 \tag{3}$$

$$7 \times 2 = 14 \tag{4}$$

Q 1. Write an equivalent expression to each expression below by using a commutative property.
$5 + ab$

Q 2. Why $ab = ba$?

5.3 Associative Properties

Numbers can be grouped in any order for addition or multiplication.
$a + (b + c) = (a + b) + c$
$a \cdot (b \cdot c) = (a \cdot b) \cdot c$

Condition: Addition and multiplication; for three or more numbers or terms
Action: **regroup** by using parentheses to change the operation order

$$(3 + 6) + 2 = 9 + 2 = 11 \tag{5}$$

$$3 + (6 + 2) = 3 + 8 = 11 \tag{6}$$

$$(2 \times 7) \times 4 = 14 \times 4 = 56 \tag{7}$$

$$2 \times (7 \times 4) = 2 \times 28 = 56 \tag{8}$$

Q 3. Why $a \cdot (b \cdot c) = (a \cdot b) \cdot c$?

5.4 Distributive Property

Condition: multiplication over addition;
Action: **split** a large multiplication problem into two smaller ones and add the results together without changing the final result.

$$17 \times 101 = 1,717 \tag{9}$$

$$17 \times 101 = 17 \times (100 + 1) = 17 \times 100 + 17 \times 1 = 1,700 + 17 = 1,717 \tag{10}$$

5.5 Identity Properties

$0 + a = a$
0 is called the addictive identity.
$1 \cdot a = a$
1 is called the multiplicative identity

Idea: any number can be regarded as a number and some operations or the result of some operation.

$24 = 2 \times 10 + 4 \times 1$

-6: means 6 apples are taken away or you owe sb. 6 dollars.