

Fraction division: Invert and Multiply

Five approaches to make sense of Invert and Multiply

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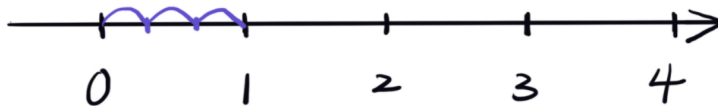
August 2021

$$\frac{1}{4} \div 2 = \frac{1}{8} \quad (1)$$



$$\begin{aligned} \frac{1}{4} \div 2 &= \frac{1}{4} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned} \quad (2)$$

$$4 \div \frac{1}{3} = 12 \quad (3)$$



$$\begin{aligned} 4 \div \frac{1}{3} &= 4 \times 3 \\ &= 12 \end{aligned} \tag{4}$$

Approach 1: think about $\times(1 \div \textit{fraction})$ which is reciprocal

$$\begin{aligned} \frac{1}{4} \div 2 &= \frac{1}{4} \times 1 \div 2 \\ &= \frac{1}{4} \times (1 \div 2) \\ &= \frac{1}{4} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned} \tag{5}$$

Another example

$$\begin{aligned} \frac{3}{4} \div \frac{1}{2} &= \frac{3}{4} \times 1 \div \frac{1}{2} \\ &= \frac{3}{4} \times (1 \div \frac{1}{2}) \\ &= \frac{3}{4} \times 2 \\ &= \frac{3}{2} \end{aligned} \tag{6}$$

Approach 2: fraction is equal to division

$$\begin{aligned}\frac{1}{4} \div 2 &= (1 \div 4) \div 2 \\ &= 1 \div (4 \times 2) \\ &= 1 \div 8 \\ &= \frac{1}{8}\end{aligned}\tag{7}$$

$$\begin{aligned}4 \div \frac{1}{3} &= 4 \div (1 \div 3) \\ &= 4 \div 1 \times 3 \\ &= 12\end{aligned}\tag{8}$$

$$\begin{aligned}6 \div 3 &= \frac{6}{3} \\ &= \frac{6 \times 1}{3} \\ &= 6 \times \frac{1}{3} \text{ (reciprocal } 3 \times \frac{1}{3} = 1) \\ &= 2\end{aligned}\tag{9}$$

Q 1. Mom plans to cut each bagel into 3 pieces to make sandwich.

1. How many sandwiches can she make if she has 2 dozens of bagels?
2. Joyce has 48 guests for her bagel sandwich party. How many bagels will mom need to cut?

Approach 3: property of division

$$m \div n = \frac{m}{n} = \frac{m \times k}{n \times k} = \frac{m \div k}{n \div k} \quad (10)$$

$\frac{m}{n} = \frac{m}{k} \div \frac{n}{k}$	(11)
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$$\begin{aligned} \frac{a}{b} \div \frac{c}{d} &= \left(\frac{a}{b} \times d\right) \div \left(\frac{c}{d} \times d\right) \\ &= \left(\frac{a}{b} \times d\right) \div c \\ &= \left(\frac{a}{b} \times d \times \frac{1}{c}\right) \div \left(c \times \frac{1}{c}\right) \\ &= \frac{a}{b} \times \frac{d}{c} \div 1 \\ &= \frac{a}{b} \times \frac{d}{c} \end{aligned} \quad (12)$$

Approach 4: use relationship between multiplication and division

$$\begin{aligned} m \div n &= k \\ m &= nk \end{aligned} \quad (13)$$

$$\begin{aligned} \frac{a}{b} \div \frac{c}{d} &= \frac{x}{y} \\ \frac{a}{b} &= \frac{c}{d} \cdot \frac{x}{y} \\ \frac{d}{c} \cdot \frac{a}{b} &= \frac{d}{c} \cdot \frac{c}{d} \cdot \frac{x}{y} \\ \frac{d}{c} \cdot \frac{a}{b} &= \frac{x}{y} \\ \frac{x}{y} &= \frac{d}{c} \cdot \frac{a}{b} \\ \frac{x}{y} &= \frac{a}{b} \cdot \frac{d}{c} \end{aligned} \quad (14)$$

Approach 5: similar denominator

$$\frac{2}{3} \div \frac{1}{4} = \frac{8}{12} \div \frac{3}{12} = \frac{8 \times \frac{1}{12}}{3 \times \frac{1}{12}} = \frac{8}{3} \quad (15)$$