1 Exponents

1. what components do an exponent number have?

$$a^n$$

a - base ; n - power

2. The meaning of an exponent

$$a^n = \underbrace{a \cdots a}_{\text{n}}$$

$$\text{Ex. } 5^3 = \underbrace{5 \cdot 5 \cdot 5}_{3}$$

3. What kind of numbers can the base be? What kind of numbers can the power be?

a: any real number (your teacher might say rational number)
n: any rational numbers

4. **Check if the bases and or powers are the same** when doing operations over exponents.

2 Operations

$$x^a \times x^b = x^{a+b} \quad (1)$$

$$a^n \cdot b^n = (ab)^n \quad (2)$$

$$\frac{x^a}{x^b} = x^{a-b} \quad (3)$$

$$(\frac{1}{a})^n = a^{-n} \quad (4)$$

$$\text{Reciprocal: } \frac{1}{x^n} \cdot x^n = x^{-n} \cdot x^n = 1 \quad (5)$$

$$(x^n)^m = x^{nm} \quad (6)$$

$$\text{If } \sqrt{a} = a^{\frac{1}{2}} = b, \text{then } b^2 = a \quad (7)$$

$$\sqrt[n]{a^m} = a^{\frac{m}{n}} \quad (8)$$

$$a^0 = 1 \text{ where } a \neq 0 \quad (9)$$

$$\begin{aligned} &0^0 \text{ is not defined.} \\ &x^n : \text{if } x = 0, \text{then } n > 0 \end{aligned} \quad (10)$$

3 Memorize some squares and their roots

$$\sqrt{1} = \tag{11}$$
$$\sqrt{4} = \tag{12}$$
$$\sqrt{9} = \tag{13}$$
$$\sqrt{16} = \tag{14}$$
$$\sqrt{25} = \tag{15}$$
$$\sqrt{36} = \tag{16}$$
$$\sqrt{49} = \tag{17}$$
$$\sqrt{64} = \tag{18}$$
$$\sqrt{81} = \tag{19}$$
$$\sqrt{100} = \tag{20}$$

4 Scientific Notations

$$\boxed{N \times 10^a \text{ where } 1 \leq N10} \tag{21}$$

Q 1. $12,450 = 1.2450 \times ?$

Q 2. $0.00236 = 2.36 \times ?$

Q 3. $4.58 \times 10^4 =$

Q 4. $3.4 \times 10^{-2} =$

Q 5. $(3.0 \times 10^5)(4.1 \times 10^{-3}) =$

Q 6. $\frac{2.5 \times 10^{-7}}{5 \times 10^6} =$

5 Multiplying

$$\boxed{x^a \times x^b = x^{a+b}} \tag{1}$$

Ex. $2^3 \times 2^4 = \underbrace{2 \cdot 2 \cdot 2}_{3} \times \underbrace{2 \cdot 2 \cdot 2 \cdot 2}_{4} = 2^{3+4}$

Proof:

$$\begin{aligned} \because x^a &= \underbrace{x \cdots x}_{\text{a}} \\ x^b &= \underbrace{x \cdots x}_{\text{b}} \\ \therefore x^a \times x^b &= \underbrace{x \cdots a}_{\text{a}} \times \underbrace{a \cdots b}_{\text{b}} \\ &= \underbrace{x \cdots \cdots x}_{\text{a+b}} \\ &= x^{a+b} \end{aligned}$$

Comment: **base must be the same.**

Q 1. How are exponent related to multiplication?

$$2^4 =$$

Q 2. how many zeros are in the answer? Is it same as the power? (using , to separate every 3 digits)

$$10^8 =$$

Q 3. Write the expressions without exponents

$$3^3x^2y^4z^6 =$$
$$(a+b)^3 =$$

Q 4. $2^3 \cdot 2^9 =$

Q 5. $a^3 \cdot a^9 =$

Q 6. $2^a \cdot 2^9 =$

Q 7. $3^2 \cdot 2^2 \cdot 3^3 \cdot 2^4 =$

Q 8. $4 \cdot x^6 \cdot y^5 \cdot x^4 \cdot y =$

$$a^n \cdot b^n = (ab)^n \tag{2}$$

Ex. $2^3 \cdot 5^3 = \underbrace{2 \cdot 2 \cdot 2}_{3} \times \underbrace{5 \cdot 5 \cdot 5}_{3} = (2 \times 5)^3$

Proof:

$$\begin{aligned} \because a^n &= \underbrace{a \cdots a}_{\text{n}} \\ b^n &= \underbrace{b \cdots b}_{\text{n}} \\ \therefore a^n \times b^n &= \underbrace{a \cdots a}_{\text{n}} \times \underbrace{b \cdots b}_{\text{n}} \\ &= \underbrace{(ab) \cdots (ab)}_{\text{n}} \\ &= (ab)^n \end{aligned}$$

- Comment 1: All have the same power n.
- Comment 2: commutative property of multiplication. $a \cdot b = b \cdot a$
- Comment 3: associative property of multiplication. $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

Q 9. $a^6 \cdot b^6 =$

Q 10. $4^6 \cdot 7^6 =$

Q 11. $2^6 \cdot x^6 \cdot 7^6 =$

6 Dividing

$$\frac{x^a}{x^b} = x^{a-b} \tag{3}$$

Ex. $2^3 \div 2^1 = \frac{2^3}{2^1} = \frac{2 \cdot 2 \cdot 2}{2} = (2)^{3-1} = 2^2$

Proof:

$$\begin{aligned} \because x^a &= \underbrace{x \cdots x}_{\text{a}} \\ x^b &= \underbrace{x \cdots x}_{\text{b}} \\ \therefore \frac{\overbrace{x \cdots x}^{a}}{\underbrace{x \cdots x}_{b}} &= \underbrace{x \cdots x}_{a-b} \\ &= x^{a-b} \end{aligned}$$

Comment: $x \neq 0$

Q 1. $\frac{x}{x} =$

Q 2. $\frac{x^2}{x} =$

Q 3. $\frac{x^3}{x^2} =$

Q 4. $\frac{x^1}{x^2} =$

Q 5. $\frac{x^2}{x^3} =$

Q 6. $2^{10} \div 2^6 =$

Q 7. $\frac{4x^6y^3z^2}{2x^4y^3z} =$

7 Testing the power of zero

$$a^0 = 1 \text{ where } a \neq 0 \qquad (4)$$

Ex. $5^0 = 5^{2-2} = \frac{5^2}{5^2} = 1$

Proof:

$$\begin{aligned} a^0 &= 1 \text{ where } a \neq 0 \\ a^0 &= a^{n-n} \\ &= \frac{a^n}{a^n} \\ &= 1 \end{aligned}$$

Q 1. $m^2 \div m^2 =$

Q 2. $\frac{4x^3y^4z^7}{2x^3y^3z^7} =$

Q 3. $\frac{(2x^2+3x)^4}{(2x^2+3x)^4} =$

8 Working with negative exponents

$division \rightarrow fraction\ and\ decimal \rightarrow negative\ exponent$

$$(\frac{1}{a})^n = a^{-n} \tag{5}$$

Ex. $(\frac{1}{5})^2 = \frac{1}{5} \times \frac{1}{5} = \frac{1^2}{5^2} = \frac{5^0}{5^2} = 5^{0-2} = 5^{-2}$

Ex. Change division into multiplication:
$\frac{5^2}{5^3} = (5^2) \cdot (5^{-3}) = 5^{2-3} = 5^{-1} = \frac{1}{5}$

Proof: $(\frac{1}{a})^n = \frac{1^n}{a^n} = \frac{a^0}{a^n} = a^{0-n} = a^{-n}$

- Comment 1: $a^0 = 1$
- Comment 2: change a division problem into a multiplication problem.

Q 1.

$$5^{-1} =$$

Q 2.

$$5^{-3} =$$

Q 3.

$$\frac{a^n}{a^m} = a^n \cdot a^{-m}$$

$$\text{Reciprocal:} \quad \frac{1}{x^n} \cdot x^n = x^{-n} \cdot x^n = 1 \qquad (6)$$

Ex. $\frac{1}{5^2} \cdot (5^2) = 5^{-2} \cdot 5^2 = 5^{2-2} = 5^0 = 1$

Q 4. $2^{-3} =$
The reciprocal of 2^3 is _

Q 5. $6^{-1} =$
The reciprocal of 6 is _

Q 6. $z^{-3} =$
The reciprocal of z^3 is _

Q 7. $\frac{1}{3^{-4}} = \frac{1}{\frac{1}{3^4}} = 1 \div \frac{1}{3^4} =$

Q 8. $\frac{x^2y^3}{3z^{-4}} =$

Q 9. $\frac{4a^3b^5c^6d}{a^{-1}a^{-2}} =$

9 Power of powers

$$(x^n)^m = x^{nm} \tag{7}$$

Ex.

$$(5^2)^3 = \underbrace{5^2 \cdot 5^2 \cdot 5^2}_{3} = \underbrace{\overbrace{5 \cdot 5}^{2} \cdot \overbrace{5 \cdot 5}^{2} \cdot \overbrace{5 \cdot 5}^{2}}_{3} = \underbrace{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}_{2\times 3}$$

Proof: $\underbrace{x^n \cdots x^n}_{m} = \underbrace{\overbrace{x \cdots x}^{n} \cdots \overbrace{x \cdots x}^{n}}_{m} = \underbrace{x \cdots \cdots x}_{nm}$

Q 1. $(6^{-3})^4 =$

Q 2. $(3^2)^{-5} =$

Q 3. $(x^{-2})^{-3} =$

Q 4. $(3x^2y^3)^2 =$

Q 5. $(3x^{-2}y)^2(2xy^{-3})^4 =$

Q 6. $(x^2y^3)^{-2}(x^{-2}y^{-3})^{-4} =$

10 Radical: a non-binary operation

Power is a fraction

$$\text{If } \sqrt{a} = a^{\frac{1}{2}} = b, \text{then } b^2 = a \tag{8}$$

Proof:

$$\because b = \sqrt{a} = a^{\frac{1}{2}}$$

$\therefore b^2 = (\sqrt{a})^2 = (a^{\frac{1}{2}})^2 = a^{\frac{1}{2}\times 2} = a^1 = a$

$$\sqrt[n]{a^m} = a^{\frac{m}{n}} \tag{9}$$

Q 1. $\sqrt{2} \cdot \sqrt{3} =$

Q 2. $\sqrt{8} \div \sqrt{4} =$

Q 3. $\sqrt{2} + \sqrt{3} =$

Q 4. $\sqrt{3} \cdot \sqrt[3]{3} =$

Q 5. $4\sqrt{3} + 2\sqrt{3} =$

Q 6. $m\sqrt{a} + n\sqrt{a} =$

Q 7. $m\sqrt{a} - n\sqrt{a} =$

Q 8. $\sqrt{a}\sqrt{a} =$

Q 9. $\sqrt{a}\sqrt{b} =$

Q 10. $\frac{\sqrt{a}}{\sqrt{b}} =$

Q 11. $6x^2 \cdot \sqrt[3]{x} =$

Q 12. $3\sqrt{x} \cdot \sqrt[4]{x^3} =$

Q 13. $4\sqrt{x} \cdot \sqrt[3]{a} =$

11 Scientific Notation

Q 1. Comparing with exponents (Hint: write big numbers in scientific notation, then compare.)

$$943,260,000,000,000,000,000,000 =$$

$$8,720,000,000,000,000,000,000,000 =$$