Distribution

PH: Chapter 2-7, 2-8

November 3, 2021

- A **term** is made up of variable(s) and/or number(s) joined by multiplication and division. Terms are separated from one another by addition or subtraction.
- You can freely group terms together and treat these term groups (clusters) as a single term when you need.

Question 1. In expression $(a+b)^2 + 3ab + 9$, find each term.

1 distributing / unfactoring

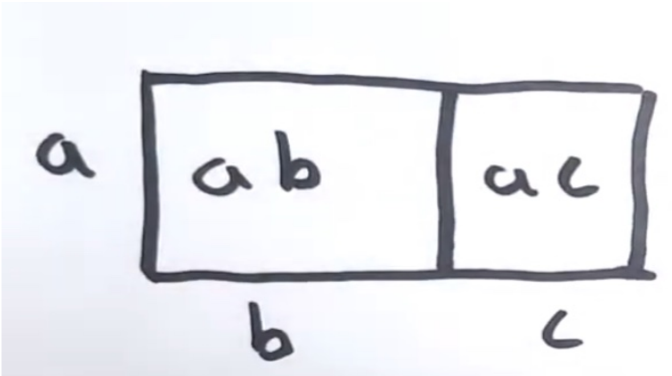
- distributing / Unfactoring – Distributing a common thing to each. e.g. distributing is to use in or as an operation so as to be mathematically distributive. Multiplication distributes over addition. Addition is not distributed over multiplication. Being an operation that produces the same result when operating on the whole mathematical expression as when operating on each part and collecting the results.
- Factoring – taking out a common thing from each
- must be able to do distributing and factoring very well.

Caution! The above understand of distributing is wrong: "giving one to each or distributing a common thing to each" refers to addition not multiplication. e.g. Grace has 4 candies and Joyce 7 candies. Now I distribute 5 to each. The math concept of "distributing" is different.

1.1 Proof of distributive property of multiplication over addition

$$\begin{aligned} a \times (b+c) &= ab + ac \\ a \times (b+c) &= \underbrace{(b+c) + (b+c) + \cdots + (b+c)}_{\text{a}} \\ &= \underbrace{b + b + \cdots + b}_{\text{a}} + \underbrace{c + c + \cdots + c}_{\text{a}} \\ &= ab + ac \end{aligned} \tag{1}$$

Proof with geometry:



Area of the rectangular: $a(b+c) = ab + ac$

1.2 Prentice Hall (PH) 2.7: Distributive property of multiplication over addition

$$a \times (b+c) = ab + ac \textbf{ distributing} \tag{2}$$

$$(b+c) \times a = ba + ca. \textbf{ commutative property of multiplication} \tag{3}$$

$$ab + ac = a \times (b+c) \textbf{ factoring} \tag{4}$$

Generalizing to unlimited number of terms.

$$a \times (b+c+d+e+...) = ab + ac + ad + ae + ... \tag{5}$$

$$a \times (b+c+d+e+...) = ab + ac + ad + ae + ... \tag{6}$$

$$ab + ac + ad + ae + ... = a \times (b+c+d+e+...) \tag{7}$$

Question 1. $2(4x + 3y - 6) =$

Question 2. $67 \times 102 = 67 \times (100+2) = 67 \times 100 + 67 \times 2$

Question 3. $17 \times 102 = 17 \times (100+2) = 17 \times 100 + 17 \times 2$

- positive number = addition; negative number = subtraction
- Number's expanded form
- any number = addition (such as expanded form) or a result by applying different; such as $4 = 2^2$ operations on different numbers
- Number/terms is identical to operations

Look at the following two questions to see if you'd better do distributing first or adding first.

Question 4. $60(\frac{1}{2} + \frac{3}{5} - \frac{3}{4} + \frac{13}{15}) =$

Question 5. $43(160 - 159 + 433 - 432) =$

Question 6. $a(a^4 + 2a^2 + 3) =$

Question 7. $a^4(2a^2 - 3a^{-2} + a^{-4} + 5a^{\frac{1}{3}}) =$

---

2 Distributing Signs

2.1 2.7: Distributive property of multiplication over subtraction

$$a \times (b - c) = ab - ac \textbf{ unfactoring} \tag{8}$$

$$(b - c) \times a = ba - ca. \textbf{ commutative property of multiplication} \tag{9}$$

$$ab - ac = a \times (b - c) \textbf{ factoring} \tag{10}$$

2.2 Collecting like terms

Think an expression with only addition signs by associating the signs with the term/number behind them. This is very important. This way of thinking can help you dealing with negative numbers in all sorts of operations consistently. You only think "-" as a negative sign not an operation sign.

$$3x - 4y + 2z = 3x + (-4y) + 2z \tag{11}$$

2.3 Inverse of a sum

$$-1 \cdot a = -a \tag{12}$$

$$-1 \cdot (a + b) = -a + (-b) = -a - b \tag{13}$$

$$\begin{aligned} -1 \cdot (a - b) &= -1 \cdot [a + (-b)] \\ &= (-1) \times (a) + (-1) \times (-b) \\ &= -a + b \end{aligned} \tag{14}$$

Question 1. $+2(4x + 2y - 3z + 7) =$

Question 2. $-2(4x + 2y - 3z + 7) =$

Question 3. $4x(x - 2) - (5x + 3) =$

Reversing the roles in distributing based on the commutative property of multiplication

$$
\begin{aligned}
(a+b) \times \boxed{\text{(c+d)}} &= a(c+d) + b(c+d) \\
&= ac + ad + bc + bd
\end{aligned} \tag{15}
$$

$$
\begin{aligned}
\boxed{\text{(a+b)}} \times (c+d) &= (a+b)c + (a+b)d \\
&= ac + bc + ad + bd
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
(a+b+c+\cdots) \times (x+y+z+\cdots) =& a(x+y+z+\cdots) \\
& + b(x+y+z+\cdots) \\
& + c(x+y+z+\cdots) \\
& + \cdots \\
=& ax + ay + az + \cdots + bx + by + bz + \cdots + cx + cy + cz + \cdots + \cdots
\end{aligned} \tag{17}
$$

$$
\begin{aligned}
(a+b)^2 &= (a+b) \times (a+b) \\
&= a^2 + ba + ab + b^2 \\
&= a^2 + 2ab + b^2
\end{aligned} \tag{18}
$$

Question 4. $(a-b)^2 =$

Question 5. $(a-b)(a+b) =$

Question 6. $(x^2+1)(y-2) =$

Question 7. $(x+3)^2 =$

Question 8. $(a+b)^3 =$

Question 9. $(a-b)^3 =$

Question 10. $(a+b)(a^2-ab+b^2) =$

Question 11. $(a-b)(a^2+ab+b^2) =$

Question 12. Palindromes: 121, 14,641 (when you reverse the digits of a number, it yields the same number.)
You can get a palindrome by reversing the digits of any number and then adding the reversal to the original number.
146 + 641 = 787