

# Math Tutor

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August 2021

Exponents - operations

## 1 Multiplying

$$x^a \times x^b = x^{a+b} \tag{1}$$

Comment 1:  $\underbrace{x \dots x}_a \times \underbrace{x \dots x}_b = \underbrace{x \dots x}_{a+b}$

comment 2: base must be the same.

**Question 1.**  $2^3 \cdot 2^9 =$

**Question 2.**  $a^3 \cdot a^9 =$

**Question 3.**  $2^a \cdot 2^9 =$

**Question 4.**  $3^2 \cdot 2^2 \cdot 3^3 \cdot 2^4 =$

**Question 5.**  $4 \cdot x^6 \cdot y^5 \cdot x^4 \cdot y =$

$$a^n \cdot b^n = (ab)^n \tag{2}$$

Comment 1:  $\underbrace{a \dots a}_n \times \underbrace{b \dots b}_n = \underbrace{ab \dots ab}_n$

Comment 2: commutative property of multiplication.  $a \cdot b = b \cdot a$

**Question 6.**  $4^8 \cdot 7^6 =$

## 2 Dividing

$$\frac{x^a}{x^b} = x^{a-b} \quad (3)$$

Comment 1:  $\frac{\overbrace{x \dots x}^a}{\underbrace{x \dots x}_b} = \underbrace{x \dots x}_{a-b}$

Comment 2:  $x \neq 0$

Question 1.  $2^{10} \div 2^6 =$

Question 2.  $\frac{4x^6y^3z^2}{2x^4y^3z} =$

## 3 Testing the power of zero

$$a^0 = 1 \text{ where } a \neq 0 \quad (4)$$

Question 1.  $m^2 \div m^2 =$

Question 2.  $\frac{4x^3y^4z^7}{2x^3y^3z^7} =$

Question 3.  $\frac{(2x^2+3x)^4}{(2x^2+3x)^4} =$

## 4 Working with negative exponents

*division*  $\rightarrow$  *fraction* and *decimal*  $\rightarrow$  *negative exponent*

$$\left(\frac{1}{a}\right)^n = a^{-n} \quad (5)$$

Comment: change a division problem into a multiplication problem.

**Question 1.**  $\frac{a^n}{a^m} = a^n \cdot a^{-m}$

Reciprocal:  $\frac{1}{x^n} \cdot x^n = x^{-n} \cdot x^n = 1 \quad (6)$

**Question 2.**

$$2^{-3} =$$

The reciprocal of  $2^3$  is \_

**Question 3.**  $6^{-1} =$

The reciprocal of 6 is \_

**Question 4.**  $z^{-3} =$

The reciprocal of  $z^3$  is \_

**Question 5.**  $\frac{1}{3^{-4}} = \frac{1}{\frac{1}{3^4}} = 1 \div \frac{1}{3^4} =$

**Question 6.**  $\frac{x^2 y^3}{3z^{-4}} =$

**Question 7.**  $\frac{4a^3 b^5 c^6 d}{a^{-1} a^{-2}} =$

## 5 Power of powers

$$(x^n)^m = x^{nm} \quad (7)$$

Comment:  $\underbrace{x^n \dots x^n}_m = \underbrace{\overbrace{x \dots x}^n \dots \overbrace{x \dots x}^n}_m = \underbrace{x \dots \dots x}_{nm}$

**Question 1.**  $(6^{-3})^4 =$

**Question 2.**  $(3^2)^{-5} =$

**Question 3.**  $(x^{-2})^{-3} =$

**Question 4.**  $(3x^2y^3)^2 =$

**Question 5.**  $(3x^{-2}y)^2(2xy^{-3})^4 =$

**Question 6.**  $(x^2y^3)^{-2}(x^{-2}y^{-3})^{-4} =$

## 6 Radical: a non-binary operation

Power is a fraction

$$\text{If } \sqrt{a} = a^{\frac{1}{2}} = b, \text{ then } b^2 = a \quad (8)$$

$$\sqrt[n]{a^m} = a^{\frac{m}{n}} \quad (9)$$

**Question 1.**  $\sqrt{2} \cdot \sqrt{3} =$

**Question 2.**  $\sqrt{8} \div \sqrt{4} =$

**Question 3.**  $\sqrt{2} + \sqrt{3} =$

**Question 4.**  $\sqrt{3} \cdot \sqrt[3]{3} =$

**Question 5.**  $4\sqrt{3} + 2\sqrt{3} =$

**Question 6.**  $m\sqrt{a} + n\sqrt{a} =$

**Question 7.**  $m\sqrt{a} - n\sqrt{a} =$

**Question 8.**  $\sqrt{a}\sqrt{a} =$

**Question 9.**  $\sqrt{a}\sqrt{b} =$

**Question 10.**  $\frac{\sqrt{a}}{\sqrt{b}} =$

**Question 11.**  $6x^2 \cdot \sqrt[3]{x} =$

**Question 12.**  $3\sqrt{x} \cdot \sqrt[4]{x^3} =$

**Question 13.**  $4\sqrt{x} \cdot \sqrt[3]{a} =$

Memorize some squares and their roots

$$\sqrt{1} = \tag{10}$$

$$\sqrt{4} = \tag{11}$$

$$\sqrt{9} = \tag{12}$$

$$\sqrt{16} = \tag{13}$$

$$\sqrt{25} = \tag{14}$$

$$\sqrt{36} = \tag{15}$$

$$\sqrt{49} = \tag{16}$$

$$\sqrt{64} = \tag{17}$$

$$\sqrt{81} = \tag{18}$$

$$\sqrt{100} = \tag{19}$$